notes for...

FINANCIAL MATHEMATICS 12 MARCH 2014

Lesson Description

In this lesson we:

- Make use of logarithms to calculate the value of n, the time period, in the equation

$$A = P(1+i)^n \text{ or } A = P(1-i)^n.$$

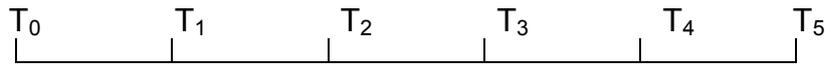- Solve problems involving present value and future value annuities.

Summary

Simple Interest	Compound Interest
The formula which helps us to calculate the accumulated amount (future value) if an original amount is invested or loaned for *n* years at a rate of *r* % simple interest is given by: $$A = P(1+i.n)$$	The formula which helps us to calculate the accumulated amount (future value) if an original amount is invested or loaned for *n* years at a rate of *r* % compound interest is given by: $$A = P(1+i)^n$$
P = present value of the investment or loan (original amount at the beginning)	**P** =present value of the investment or loan (original amount at the beginning)
A = accumulated amount or future value of the investment or loan after *n* periods	**A** =accumulated amount or future value of the investment or loan after *n* periods
n =time period	**n** =time period
i = $\dfrac{r}{100}$ for the simple interest rate *r* % (*i* is in decimal form)	**i** = $\dfrac{r}{100}$ for the simple interest rate *r* % (*i* is in decimal form)
FUTURE VALUE ANNUITY	**PRESENT VALUE ANNUITIES**
$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$ $where:$ $x = \text{payment per period}$ $n = \text{number of payments}$ $i = \text{interest rate}$	$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$ $where:$ $x = \text{payment per period}$ $n = \text{number of payments}$ $i = \text{interest rate}$

learn more, learn xtra!

LEARN XTRA IS PROUDLY BROUGHT TO YOU BY MINDSET

mindset

Timeline

$$T_0 \qquad T_1 \qquad T_2 \qquad T_3 \qquad T_4 \qquad T_5$$

Sinking Fund

- The **book value** of an asset is its value after depreciation has been taken into account.
- The **scrap value** is the book value of an asset at the end of its useful life.
- The **sinking value** is the fund set up to replace an asset at the end of its useful life.

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company will have, well in advance, set up an annuity called a **sinking fund**. A sinking fund is nothing more than a normal savings plan or future value annuity.

The proceeds from the sinking fund and the sale of the old equipment are then used to purchase the new equipment.

Sinking Fund = Cost of new machine   –   Scrap value

Cost of New Machine	Scrap value
Inflation	Depreciation on reducing balance basis
$A = P(1+i)^n$	$A = P(1-i)^n$

Balance of Loan

Balance = Loan (Capital + interest accrued over n years) – repayments for n years (together with interest)

$$\text{Balance} = P(1+i)^n - \frac{x\left[(1+i)^n - 1\right]}{i}$$

Test Yourself

Question 1

An investment of R9 000 earns 6% per annum compounded annually for a period of 4 years. Thereafter, the interest rate changes to 7% per annum compounded annually for a further 2 years. Calculate the future value of the investment at the end of the 6-year period.

A.   R13 518,69
B.   R23 008,69
C.   R13 008,69
D.   R133 008,69

Question 2

James invests R4 000 in a bank account paying 6% per annum compounded monthly. Calculate the value of his investment in 10 years time.

A.   R18 266,59
B.   R8 166,59
C.   R2 2666,50
D.   R7 277,59

Question 3

How much more money would you have over a period of 15 years on a savings of R60 000 if the interest rate of 12% per annum was compounded monthly, as compared to an interest rate of 12% compounded annually?

A.   R359 748,19

B.   R328 413,95

C.   R31 334,17

D.   no difference.

Question 4

R5 000 is deposited into a savings account. The interest rate for the first four years is 7% per annum compounded quarterly. Thereafter, the interest rate changes to 8% per annum compounded semi-annually. Calculate the value of the investment at the end of the tenth year.

A.   R10 222,23

B.   R23 645,23

C.   R10 566,28

D.   R11 456,11

Question 5

Joseph deposits x into a savings account. Two years later, he deposits a further R2   into the account. Three years after this, he deposits 3x into the account. The interest rate for the five years is 18% per annum compounded monthly. He receives R60 000 at the end of the five year period. Calculate the value of x.

   A. R6 350,31
   B. R4 452,32
   C. R5 652,30
   D. R6 230,51

Question 6

An investment of R9 000 earns 6% per annum compounded quarterly for a period of 4 years. Thereafter, the interest rate changes to 7% per annum compounded semi-annually for a further 2 years. Calculate the future value of the investment at the end of the 6-year period.

   A. R13 105,71
   B. R12 235,62
   C. R10 341,70
   D. R18 900,50

Question 7

Anna saves R6 000 a year and deposits it at the end of each year in a savings account that pays her 7% p.a. compounded annually. Find her accumulated savings immediately after the 12th deposit is made.

   A. R107 330,71
   B. R206 441,73
   C. R106 662,72
   D. R161 442,32

Question 8

Fran buys a large house with five bedrooms. She plans to occupy one room herself and let the other rooms to tenants. She expects the rental will cover the loan repayments on her house. She takes out a loan of R550 000. The interest on the loan is 8% compounded monthly. How much must she charge each tenant if the loan is to be repaid over 20 years?

   A. R 1150,11
   B. R 1261,12
   C. R 1373,12
   D. R 1000,17

Question 9

Thando dreamt of becoming a millionaire. When she was 30 years old, she invested R200 000 with an investment company. If the interest rate was 8,5% p.a. compounded monthly, how old was she when she achieved her dream? Give your answer correct to the nearest year.

   A. 50 years old
   B. 48 years old
   C. 49 years old
   D. 39 years old

Question 10

A taxi owner buys five minibuses. Each bus costs him R320 000. Determine the book value of the fleet of minibuses after four years if the depreciation is calculated at 20% p.a. on: a reducing balance.

   A. R655 369
   B. R543 345
   C. R210 453
   D. None of the above

Improve your Skills

Question 1

R3 000 is deposited into a savings account at a bank and the interest paid is 15% p.a. compound interest. After how many years will the investment be worth R5 250?

Question 2

Suppose R1 000 is deposited immediately into a savings account and continues to make monthly payments at the end of each month for a period of 5 months. Interest is 18% p.a. compounded monthly. How much money will have accumulated at the end of the 5th month?

Question 3

An investor aged 25 wishes to accumulate R10 000 000 by her 50th birthday. She will pay equal monthly payments into an account paying 15% per annum compounded monthly. Payments start on her 25th birthday and end on her 50th birthday. Find the monthly payments.

Question 4

How much can be borrowed now if a person agrees to repay a loan by means of quarterly payments of R7 000, starting in three months from now? The payments will continue for 3 years and the interest rate is 16% per annum compounded quarterly.

Question 5

A loan of R300 000 is to be repaid by means of monthly payments of R5 000, starting one month after the granting of the loan. Interest is fixed at 18% per annum compounded monthly. How long will it take to repay the loan?

Question 6

A school purchases a photocopying machine for R150 000. The machine depreciates in value at 22% p.a. on a reducing balance. The school wants to buy a new machine in 5 year's time. A new machine will cost much more in the future and its cost will escalate at 19% per annum effective. The old machine will be sold at scrap value after 5 years. A sinking fund is set up immediately in order to save up for the new machine. The proceeds from the sale of the old machine will be used together with the sinking fund and the interest earned is 14,4% p.a. compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.

a)   Find the scrap value of the old machine.

b)   Find the cost of the new machine 5 years from now.

c)   Find the amount required in the sinking fund 5 years from now.

d)   Find the equal monthly payments made into the sinking fund.

Question 7

A car is bought for R180 000. If 10% is paid in cash and the balance is paid using a bank loan. The loan is repaid over six years. The interest rate is 10,5% p.a. compounded monthly.

a)   Determine the monthly repayments.

b)   Determine the balance on the loan at the end of two years immediately after the 24th payment.